Closing Wed: HW_6A, 6B, 6C (7.4,7.5,7.7)
Closing next Wed: HW_7A, 7B, 7C (7.8,8.1)

Midterm 2 is next Thursday, May 19
Covers: 6.4, 6.5, 7.1-7.5, 7.7, 7.8, 8.1

<u>How to integrate</u>

1. Look for simplifications/substitutions
2. Products/Logs/Inverse Trig → BY PARTS
   Sin/Cos/Tan/Sec combos → TRIG
   Quadratic (under a radical) → TRIG SUB
   Rational Function → PART. FRAC.
3. If nothing seems to work, substitution.
   (u = inside, u = $\sqrt{}$, u = trig, u = e^x)

Here are four problems that don't obvously fit any of our four special methods, which means they all start with substitution!
Make a substitution in all four examples:

1. $\int e^{\sqrt{x}} dx$

2. $\int \frac{3}{x - 2\sqrt{x}} dx$

3. $\int \frac{\cos(x)}{4 - \sin^2(x)} dx$

4. $\int e^x \cos(e^x) \sin^3(e^x)\, dx$

How would you start these:

1. $\int \tan^3(x)\sec(x)\,dx$

2. $\int x^2 \ln(x)\,dx$

3 $\int x\sqrt{5-x^2}dx$

4. $\int \frac{\sqrt{x^2-1}}{x^2}dx$

5. $\int \frac{x^2+1}{x^2-2x-3}dx$

6. $\int x\tan^{-1}(x)dx$

7. $\int \frac{dx}{\sqrt{4x^2+8x-12}}dx$

7.7 Approximating Integrals: The vast majority of integrals can NOT be done with any of our methods. So we need to know approximation methods.

Recall from the first week of the quarter: **To approximate** $\boldsymbol{\int_a^b f(x)dx}$

1. Pick **n = number of subdivisions**. Compute $\Delta \boldsymbol{x} = \frac{\boldsymbol{b-a}}{\boldsymbol{n}}$.
2. Label the tick marks: $\boldsymbol{x_i = a + i\Delta x}$
3. Use an approximation method:

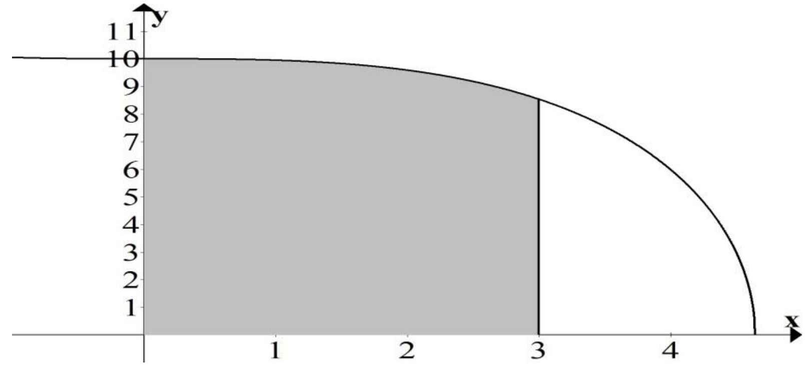
$$L_n = \Delta x[f(x_0) + f(x_1) + \cdots + f(x_{n-1})] \quad \text{(Left endpoint)}$$
$$R_n = \Delta x[f(x_1) + f(x_2) + \cdots + f(x_n)] \quad \text{(Right endpoint)}$$
$$M_n = \Delta x[f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)] \quad \text{(Midpoint)}$$

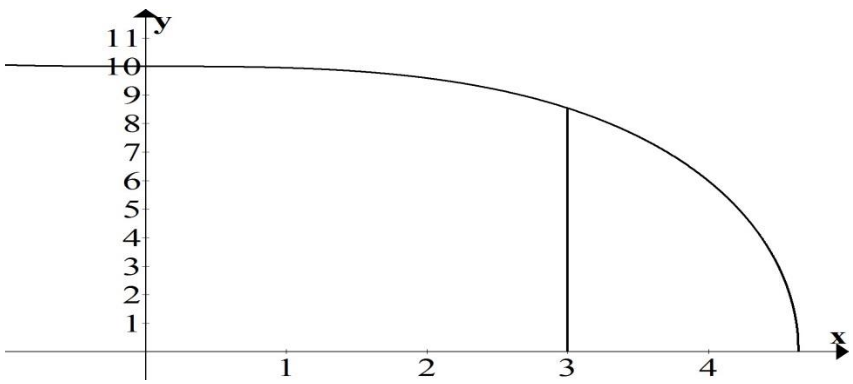
Example: Approximate $\int_0^3 \sqrt{100 - x^3}dx$ with L_3, R_3, and M_3.

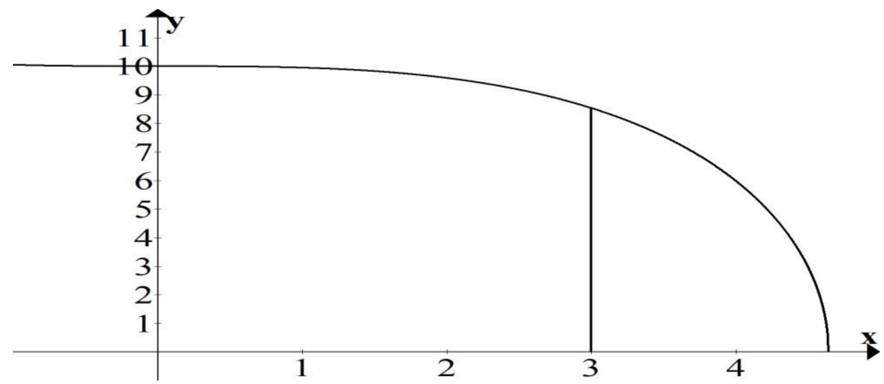
1. $\Delta x = \frac{b-a}{n} = \frac{3-0}{3} = 1$
2. $x_i = a + i\Delta x = 0 + i(1) = i$, so $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$

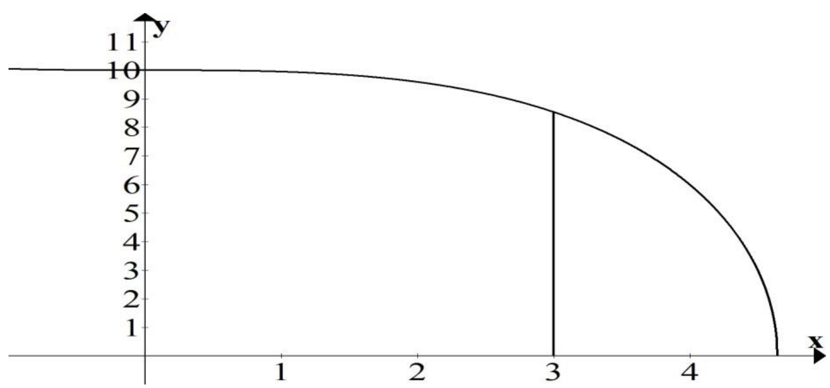
$$L_3 = (1)\left[\sqrt{100-(0)^3} + \sqrt{100-(1)^3} + \sqrt{100-(2)^3}\right] \approx 29.5415$$
$$R_3 = (1)\left[\sqrt{100-(1)^3} + \sqrt{100-(2)^3} + \sqrt{100-(3)^3}\right] \approx 28.0855$$
$$M_3 = (1)\left[\sqrt{100-(0.5)^3} + \sqrt{100-(1.5)^3} + \sqrt{100-(2.5)^3}\right] \approx 29.0091$$

$$\int_0^3 \sqrt{100 - x^3}\,dx \approx 28.9442$$









Today, we add 2 more methods:

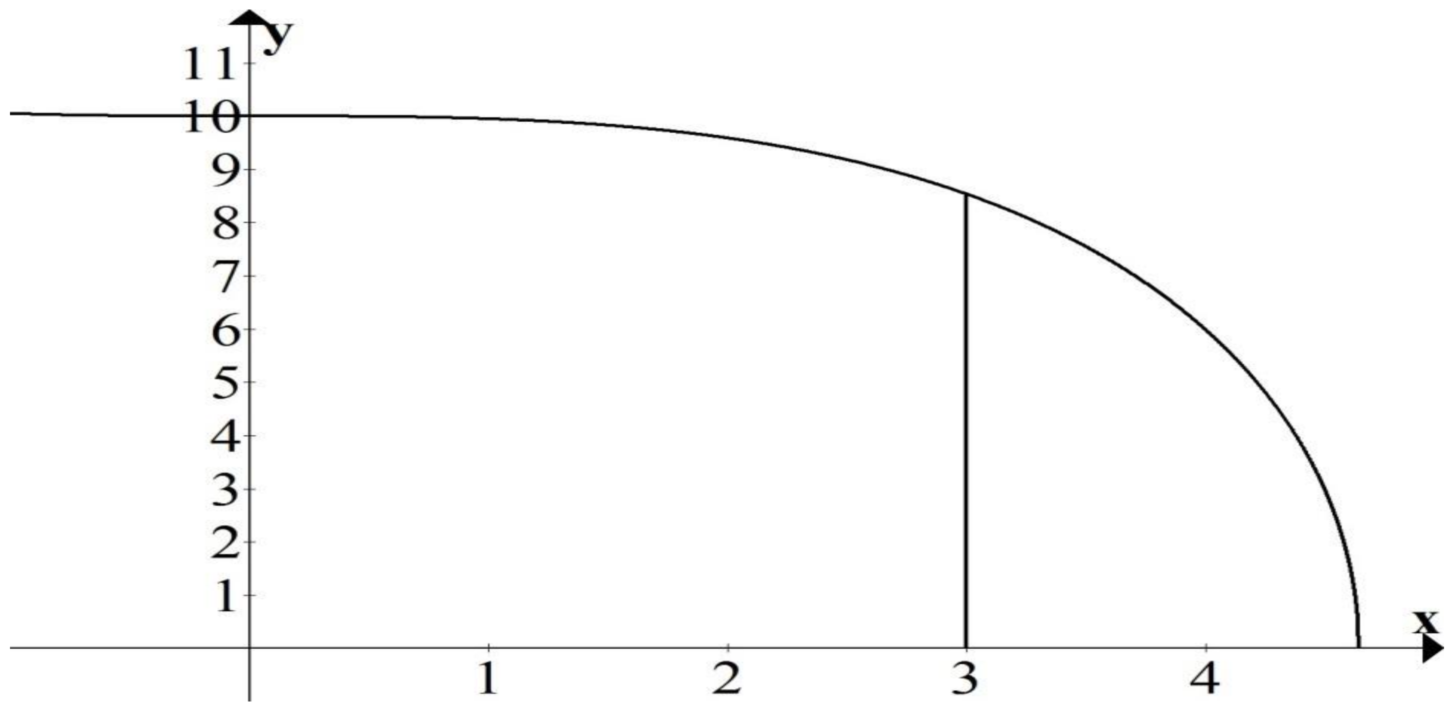
Trapezoid Rule:

$$T_n = \frac{1}{2}\Delta x[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

Note: all "middle terms" are multiplied by 2)

In our example:

$$T_3 = \frac{1}{2}(1)\left[\sqrt{100-(0)^3} + 2\sqrt{100-(1)^3} + 2\sqrt{100-(2)^3} + \sqrt{100-(3)^3}\right] \approx 28.8135$$



Simpson's Rule: (*n* must be even)

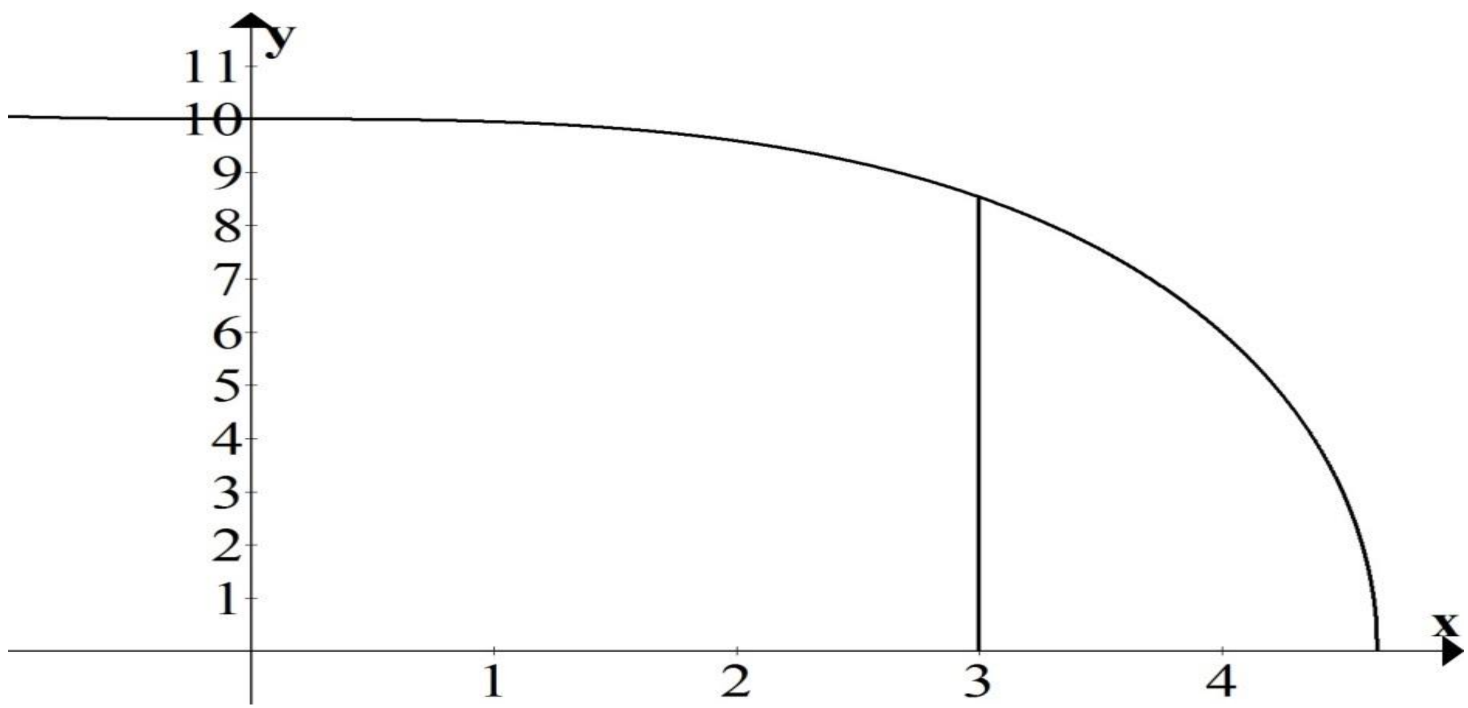
$$S_n = \frac{1}{3}\Delta x[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

Note: Alternating multiplying middle terms by 4 and 2.

In our example, with n = 6, you get: $\Delta x = \frac{3-0}{6} = \frac{1}{2}$,

$$x_0 = 0, x_1 = \frac{1}{2}, x_2 = 1, x_3 = \frac{3}{2}, x_4 = 2, x_5 = \frac{5}{2}, x_6 = 3$$

$$S_6 = \frac{1}{3}\cdot\frac{1}{2}\Big[\sqrt{100-(0)^3} + 4\sqrt{100-(0.5)^3} + 2\sqrt{100-(1)^3} + 4\sqrt{100-(1.5)^3}$$
$$+2\sqrt{100-(2)^3} + 4\sqrt{100-(2.5)^3} + \sqrt{100-(3)^3}\Big] \approx 28.9441$$



Consider

$$\int_{-1}^{1} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\, dx$$

Let’s approximate using n = 4 and all 5 methods.